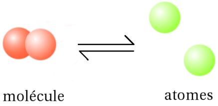
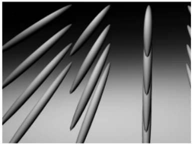
A Pure Confinement Induced Trimer in quasi-1D Atomic Waveguides





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Outline

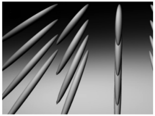
- **Context**
  - Atomic gases near *s*-wave magnetic Feshbach resonances
  - Atomic gas in low dimensions
- **2- and 3-body problem in 1D atomic waveguides**
  - Two-channel model
  - Confined induced dimers
  - Some spectrum of trimers in the vicinity of a Feshbach resonance
  - Existence of a pure Confined Induced Trimer in the 1D limit
- **Summary & perspectives**

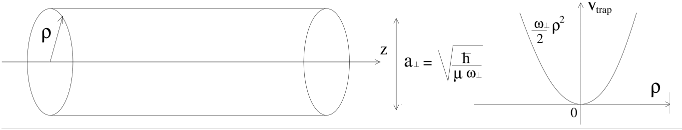
Some key features of ultracold-atoms

- Tunable effective interactions
  Magnetic Feshbach resonance $\Longrightarrow$ 3D scattering length $a(B)$
- Tunable dimensionality 3D $\leftrightarrow$ 2D, 3D $\leftrightarrow$ 1D
- Dilute limit $nb^3 \ll 1$ & possible large correlations $na^3 \gtrsim 1$ (unitary limit)

Atoms in a 1D waveguide

1D wave guide





2D isotropic harmonic trap $\Longrightarrow$ 1D atomic wave guide

1-particle energy : $E = \frac{\hbar^2 k^2}{2\mu} + \hbar\omega_\perp(2n + |m| + 1)$

- $m\hbar$: angular momentum
- n : radial quantum number
- k : 1D wavenumber

From the Efimov $-E_0 e^{-2n\pi/s_0}$ ... to the Mc Guire trimer $\frac{-4\hbar^2}{ma_{1\mathrm{D}}^2}$

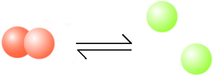
Two-channel modeling of the Feshbach resonance

Atom : $a^\dagger_{\mathbf{k}}|0\rangle$; Molecule : $b^\dagger_{\mathbf{k}}|0\rangle$

$$H = \int \frac{d^3k}{(2\pi)^3}\left[E_{\mathbf{k}} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \left(\frac{E_{\mathbf{k}}}{2} + E_{\mathrm{mol}}\right) b^\dagger_{\mathbf{k}} b_{\mathbf{k}}\right]$$

(Kinetic term & $E_{\mathrm{mol}} = E^0_{\mathrm{mol}} + \delta\mathcal{M}\mathcal{B}$)

$$+\Lambda \int \frac{d^3k d^3K}{(2\pi)^6}\left[\langle k|\delta_\epsilon\rangle b^\dagger_{\mathbf{K}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}} a_{\frac{\mathbf{K}}{2}+\mathbf{k}} + \mathrm{h.c.}\right]$$

()

$$+\frac{g}{2}\int \frac{d^3k d^3K d^3k'}{(2\pi)^9}\langle k'|\delta_\epsilon\rangle\langle\delta_\epsilon|k\rangle a^\dagger_{\frac{\mathbf{K}}{2}-\mathbf{k}'} a^\dagger_{\frac{\mathbf{K}}{2}+\mathbf{k}'} a_{\frac{\mathbf{K}}{2}+\mathbf{k}} a_{\frac{\mathbf{K}}{2}-\mathbf{k}}.$$

(atom-atom interaction)

$E_{\mathbf{k}} = \frac{\hbar^2 k^2}{2m}$ & $\langle k|\delta_\epsilon\rangle = \exp\left(-\frac{k^2\epsilon^2}{4}\right)$ cut-off function

Parameters obtained from the 2-body properties at low E

- **background scattering length:** $a_{\rm bg} \leftrightarrow g$
- **scattering length:** $a = a_{\rm bg}\left(1 - \frac{\Delta\mathcal{B}}{\mathcal{B} - \mathcal{B}_0}\right)$
- **width parameter:**

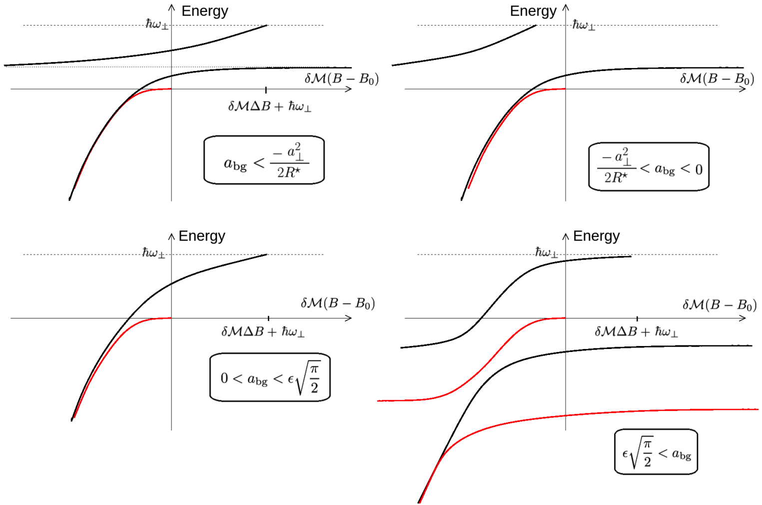
$$R^\star = \frac{\hbar^2}{m a_{\rm bg}\delta\mathcal{M}\Delta\mathcal{B}} \propto \frac{1}{\Lambda^2}$$

Petrov PRL (2004)

- **short range parameter:** $\epsilon \sim \left(\frac{mC_6}{\hbar^2}\right)^{1/4}$

Confinement Induced Dimers in 1D waveguide

- Zero-range model Olshanii PRL (1998)
- Two-channel model study:



T. Kristensen, L. Pricoupenko PRA (2015)

Study of trimers in the waveguide

- $E = E_{\rm 3body} - E_{\rm Com} < 0$
- s-wave sector
- Skorniakov Ter Martirosian equation for trimers in the 1D waveguide :

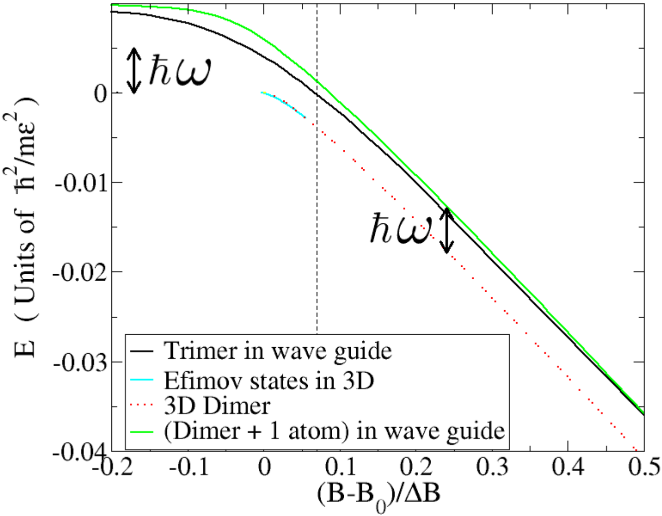
$$D(E^{\rm rel}) f(\underline{n}, \underline{m}=0, k) = 2 \sum_{\underline{n}'=0}^{\infty} \int \frac{dk'}{2\pi} \langle \underline{n}, k | \mathcal{K}(E) | \underline{n}', k' \rangle f(\underline{n}', \underline{m}'=0, k')$$

with $\quad E^{\rm rel} = E - (2\underline{n} + |\underline{m}| + 1)\hbar\omega - \frac{3\hbar^2 k^2}{4m}$

- Explore the case $a_\perp/\epsilon = 20$ for several resonances
- $n_{\rm max}$ from 100 to 400 $\Longrightarrow (2n_{\rm max}+1)^3$ values of $d^j_{m,m'}(\theta)$.
  (typical matrix sizes $\sim$ 30 000)

Example of a narrow resonance

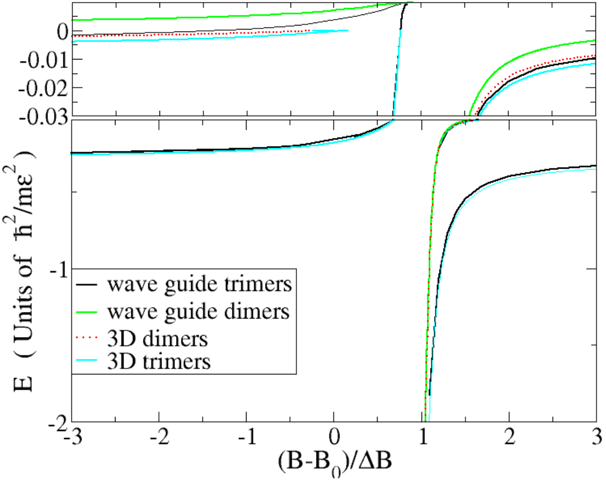
^{39}K at $B_0 = 752$ G ; $R^\star = 36.4 R_{\mathrm{vdw}}$
$a_\perp = 20\epsilon$ ($\omega = 2\pi \times 55.4$ kHz)



Broad Feshbach resonance near a shape resonance

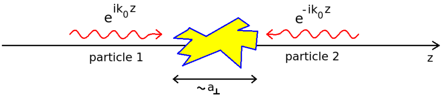
^{133}Cs at $B_0 = -12$ G ; $R^\star = 1.3 \times 10^{-3} R_{\rm vdw}$

$a_\perp = 20\epsilon$ ($\omega = 2\pi \times 6.7$ kHz)



STM equation for the purely 1D contact model

- 1D scattering length: $a_{1\mathrm{D}}$



$\langle z|\psi_{k_0}\rangle \sim e^{ik_0z} + f_{1\mathrm{D}}(k_0)e^{ik_0|z|} \qquad f_{1\mathrm{D}}(k_0) = \frac{-1}{1+ik_{1\mathrm{D}}a_{1\mathrm{D}}}$

- Lieb-Liniger interaction: $\frac{-2\hbar^2}{ma_{1\mathrm{D}}}\delta(z)$
- Dimer wavenumber: $q_\mathrm{d} = 1/a_{1\mathrm{D}} \qquad a_{1\mathrm{D}} > 0$
- 1 Trimer (Mc Guire): $q = 2q_\mathrm{d}$; $f(k) = \frac{1}{k^2+4q_\mathrm{d}^2}$

$$\left(\frac{1}{\sqrt{\frac{3k^2}{4}+q^2}} - \frac{1}{q_\mathrm{d}}\right) f(k) + 4\int \frac{dk'}{2\pi}\frac{f(k')}{q^2+k^2+k'^2+kk'} = 0$$

Quasi-1D limit for 3 atoms

- Binding wavenumber of the trimer q: $E = 2\hbar\omega - \frac{\hbar^2}{m} q^2$
- Low energy limit $q a_\perp \ll 1$

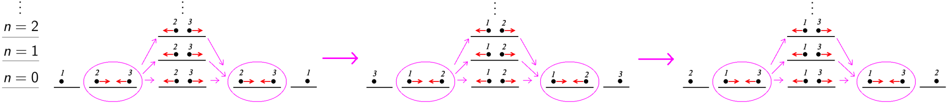
Component of the wavefunction on $n > 0$ modes can be neglected

Projection of the STM equation in the mode $n = 0$

$\Rightarrow$ Quasi-1D STM equation

$$\left(\frac{1}{\sqrt{\frac{3k^2}{4} + q^2}} - \frac{1}{q_{\mathrm{d}}} \right) f(0,0,k) + 4 \int \frac{dk'}{2\pi} \sum_{n=0}^{\infty} \frac{4^{-n} f(0,0,k')}{\frac{4n}{a_\perp^2} + q^2 + k^2 + k'^{\,2} + kk'} = 0$$

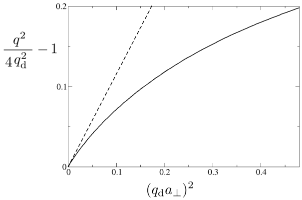
C. Mora, R. Egger, and A. O. Gogolin PR A 71, 052705 (2005)



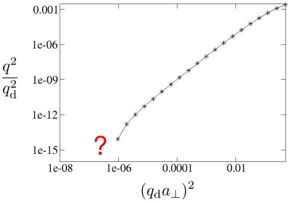
Trimer solutions in the quasi-1D limit

- Ground state: deviation from the Mc Guire trimer

$$q^2 \sim 4q_\mathrm{d}^2 \left[1 + 4(a_\perp q_\mathrm{d})^2 \ln(4/3)\right]$$



- 1 excited state : a Pure Confinement Induced Trimer



- Existence of the CIT in the 1D limit $a_\perp/a_{1\mathrm{D}} \to 0^+$?

CIT near the dimer threshold $a_\perp/a_{1D} \to 0^+$ $(\eta = 0)$

- Small parameters: $q^2 = q_{\rm d}^2(1+\chi) \quad ; \quad \eta = (qa_\perp)^2$
- Transformations: $u = \frac{k}{q}$; $\langle u|\psi\rangle = f(0,0,k)$
- Quasi-1D STM equation for $\eta \ll 1$

$$\boxed{\langle u|\mathcal{L}_0|\psi\rangle + \langle u|\delta\mathcal{L}|\psi\rangle = \sqrt{1+\chi}\langle u|\psi\rangle}$$

- $\langle u|\mathcal{L}_0|\psi\rangle = \frac{\langle u|\psi\rangle}{\sqrt{\frac{3}{4}u^2+1}} + 4\int\frac{du'}{2\pi}\frac{\langle u'|\psi\rangle}{1+u^2+u'^2+uu'}$
- $\langle u|\delta\mathcal{L}|\psi\rangle = 4\eta\ln\left(\frac{4}{3}\right)\int\frac{du'}{2\pi}\langle u'|\psi\rangle$

CIT wavefunction near the dimer threshold

- Exact Atom-dimer wave function of the Lieb-Liniger model

$$\langle u|\psi\rangle = (2\pi)\delta(u) - \frac{4}{u^2+1}$$

- Quasi-1D STM equation for small energies $\chi \to 0$

$$\Rightarrow \langle u|\psi\rangle \propto \frac{1}{\frac{3}{4}u^2+\chi} \text{ for } u \to 0$$

- CIT wavefunction: $\langle u|\psi_1\rangle = \langle u|\psi_1^{(0)}\rangle + \langle u|\delta\psi_1\rangle$

$$\langle u|\psi_1^{(0)}\rangle = \frac{\sqrt{3\chi}}{\frac{3}{4}u^2+\chi} - \frac{4}{u^2+1} \quad ; \quad \frac{\langle u|\delta\psi_1\rangle}{\langle u|\psi_1^{(0)}\rangle} \to 0 \quad \text{for} \quad \eta \to 0$$

- But . . . no quantification condition if one neglects $\langle u|\delta\psi_1\rangle$

Spectrum of the Confinement Induced Trimer

- Equation verified by $\langle u|\delta\psi_1\rangle$ at the first order in η

$$\langle u|\mathcal{L}_0 - 1|\delta\psi_1\rangle = \eta \ln\left(\frac{4}{3}\right) + \frac{4\sqrt{3\chi}}{3u^2}\left(1 - \frac{1}{(u^2+1)^2\sqrt{\frac{3}{4}u^2+1}}\right).$$

- Regular solution at $u = 0$:

$$\Longrightarrow \sqrt{\chi} = \frac{2}{\sqrt{3}} \ln\left(\frac{4}{3}\right)\eta$$

$$\Longrightarrow q^2 = q_\mathrm{d}^2\left[1 + \frac{4(a_\perp q_\mathrm{d})^4}{3}\ln^2\left(\frac{4}{3}\right)\right]$$

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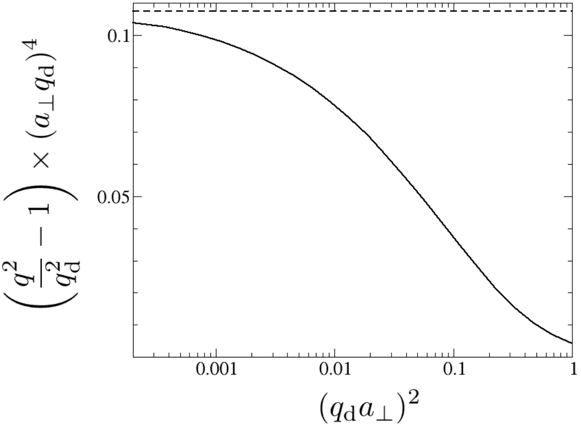

Summary & perspectives

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  - 2 quasi-1D trimers: Mc Guire & 1 Pure CIT
  - continuously connected with the lowest Efimov states

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- To be done: Pure Confinement Induced 4-body, 5-body . . . at the dimer threshold ?